JOURNAL OF COMPUTATIONAL PHYSICS 88, 467–476 (1990)

Note

The Dynamical Interactions of Cosmic Strings*

Cosmic strings [1–2] are thin, topologically stable tubes of symmetric-phase false vacuum that may have formed during a cosmological phase transition. Loops of cosmic string could have played an important role in the formation of large-scale structure in the universe. In the early universe these loops would exert a gravitational attraction on nearby matter, leading to density fluctuations that would evolve into galaxies and clusters of galaxies. An important property of cosmic strings is that for most purposes they have no inherent length scale (the width of a cosmic string is on the order of 10^{-29}cm). Thus one expects that an equal number of loops of all sizes would be formed, leading to a scale invariant spectrum of density fluctuations. This in turn leads to a scale invariant distribution of galaxies and clusters of galaxies (as measured by the galaxy–galaxy correlation function), which matches observation [3]. It is also encouraging that the amplitudes of the density fluctuations produced by string loops give realistic galaxy formation when the string tension (which is the characteristic energy scale of the string) is on the order of the symmetry-breaking scale in Grand Unified Theories (GUTs). It is difficult to imagine how scale invariant density fluctuations of the proper magnitude could have been produced in the early universe without cosmic strings. The fact that cosmic strings play an important role in a realistic model of galaxy formation is one of the main reasons for current interest in these objects.

The cosmic string scenario of galaxy formation depends crucially on the formation of loops of string. When strings are formed at the phase transition it appears that only 20% of the total string length will be in the form of loops [4], with the remainder in the form of infinitely long strings. In order to form additional loops from infinite strings it is necessary that when two strings cross they intercommute (that is, trade ends). In fact, if cosmic strings do not intercommute then the idea of their appearance in the early universe is in big theoretical trouble, for the following reason. Cosmic string loops oscillate and lose energy via gravitational radiation, so oscillating loops shrink and eventually disappear. This would explain why strings are not seen today. But infinite strings do not decay and, since the energy density of infinite strings scales roughly like non-relativistic matter, a network of non-intercommuting strings would quickly come to dominate the energy density of the universe, which of course is not observed. Thus loop formation is not just a useful feature for galaxy formation, it is required for cosmic strings to have existed at all.

* Poster presented at the Particle Astrophysics Workshop at the Lawrence Berkeley Laboratory, Berkeley, California, December 8–10, 1988.

0021-9991/90 $3.00

How strings behave when they cross depends upon their microscopic structure, so determining whether or not strings intercommute is not easy. The string is a topological defect described by a complicated system of non-linear differential equations describing a Yang–Mills gauge field coupled to a symmetry breaking Higgs field. Because these equations cannot be solved analytically we are performing a numerical simulation of the collisions of cosmic strings [5–6]. The early indications are encouraging—it appears that cosmic strings do indeed intercommute, although there are several distinct processes responsible for this. But even if strings do intercommute the details of the mechanism(s) involved may have an important influence on the distribution of loops. If strings only intercommute at certain crossing angles or at low collision velocities this could destroy the scale invariant distribution of loops.

Our computational techniques are described in detail in Ref. [6], so we present here only a brief overview. The strings we consider are vortex lines in the Abelian Higgs model, which is described by the action:

$$S = \int d^{d+1}x \left\{ -(\nabla^{\mu}\varphi)^{\dagger}(\nabla_{\mu}\varphi) - \frac{\lambda}{4}(\varphi^{\dagger}\varphi - \sigma^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right\}. \tag{1}$$

Here $\varphi(x)$ is a complex scalar field, $A_{\mu}(x)$ is the $U(1)$ gauge potential, $F_{\mu\nu} = \partial_{\mu}A_{\nu}(x) - \partial_{\nu}A_{\mu}(x)$ is the field strength, and $\nabla_{\mu}\varphi = \partial_{\mu}\varphi - ieA_{\mu}\varphi(x)$ is the gauge-covariant derivative of $\varphi(x)$. The Higgs potenial for the φ field is minimized by $|\varphi| = \sigma \neq 0$, so the vacuum of the system is degenerate and consists of the set of states where $\varphi(x) = \sigma e^{i\theta_0}$ for any fixed θ_0. Because the vacuum is non-simply connected it is possible to create topological defects (called "vortices") which cannot be deformed into the vacuum by expending any finite amount of energy. In the vortex configuration the φ field "winds" around the central maximum of the Higgs potential like $\varphi(r, \theta) \approx \sigma \exp(in\theta)$. Although the phase of φ is not constant a suitable gauge potential can be chosen such that $\varphi(x)$ is *covariantly* constant far from the vortex. Continuity of $\varphi(x)$ requires both that $\varphi(x)$ vanish at the center of the vortex and that n, the "winding number," be an integer. Away from the central part of the vortex the scalar field is locally equivalent to the vacuum, but globally there is an integral twist in the phase of $\varphi(x)$. In three dimensions the vortex extends in the third dimension to form a line-like topological defect. This is the cosmic string.

In addition to modeling cosmic strings, vortices in the Abelian Higgs model also represent tubes of magnetic flux ("Abrikosov vortices") trapped in a superconducting material in the Ginzburg–Landau theory [7–8]. In this case, however, the flux tubes are always aligned vertically with the external magnetic field, so the problem is effectively two dimensional. The general nature of the interactions between static vortices is already known [9]. When the scalar field coupling constant λ is below the critical value $\lambda = 2$ the vortices attract each other, which corresponds to a Type I superconductor. When the coupling constant is above the critical value the vortices repel, which corresponds to a Type II superconductor. At the critical value the interaction energy of two *isolated* vortices is zero, which suggested that in a collision critically coupled vortices might behave like solitons and pass right through each other. As we will show later we have found that this is not the case: critically coupled vortices interact non-trivially (they scatter at 90° in a head-on collision) and are therefore not solitons. Applied to cosmic strings this means that nearly parallel cosmic strings will intercommute. We will explain why below.

To simulate the system in a computer requires a discretization of the continuum of degrees of freedom, and since we are dealing with a gauge theory we have used techniques from lattice gauge field theory [10]. The points of space are replaced by the vertices $\mathbf{x}$ of a cubic lattice with lattice spacing a. The scalar field $\varphi(x)$ and its conjugate momentum $\pi(x)$ are represented by the variables $\varphi_{\mathbf{x}}$ and $\pi_{\mathbf{x}}$, which live on the sites of the lattice, while the gauge field $A_\mu(x)$ and its conjugate momentum (the electric field) are represented by the variables $\theta^\mu_{\mathbf{x}}$ and $E^\mu_{\mathbf{x}}$, which live on the links of the lattice. The virtue of using the lattice gauge field theory formalism is that unlike other discretization procedures it preserves the local gauge symmetry of the system. We begin the simulation with initial data describing two isolated vortices (or strings) approaching each other from a distance. This initial configuration is obtained by boosting the continuum field configuration of a stationary vortex. Once the initial configuration has been created it is propagated forward in time numerically using the equations of motion. For greater numerical stability we have used a "leapfrog" algorithm. The proper treatment of the boundary conditions is important in this kind of simulation. We have implemented both free and periodic boundary conditions (periodic up to a gauge transformation) and have specifically avoided any sort of "driven" boundary conditions, because they would not allow us to simulate a closed system with a conserved total energy. Further details of our methods are given in Ref. [6].

Figure 1 shows the collision of two critically coupled vortices in two dimensions, which represent parallel cosmic strings. We plot the total energy density as a function of position. Two isolated vortices approach each other in the x direction, collide, and form (briefly) a double wound vortex. Then two isolated vortices reemerge, but instead of passing through each other they have scattered at 90°. This shows that critically coupled vortices are not solitons.

After we performed this simulation we learned that it is possible to predict analytically the 90° scattering of critically coupled vortices in the limit of very slowly moving vortices [11]. Our simulation shows that the 90° scattering also takes place at higher velocities, and we have found the same behavior for non-critical values of the coupling constant. This non-trivial scattering is therefore apparently a generic feature of the interactions of vortices in the Abelian Higgs model.

The fact that vortices in two dimensions scatter at 90° implies that nearly parallel strings will intercommute. To see this consider Fig. 2, where two nearly parallel strings are approaching each other, and we have imagined two perpendicular planes intersecting along the line between the strings. The vertical plane bisects the angle between the crossing strings. In the horizontal plane the collision of the strings is

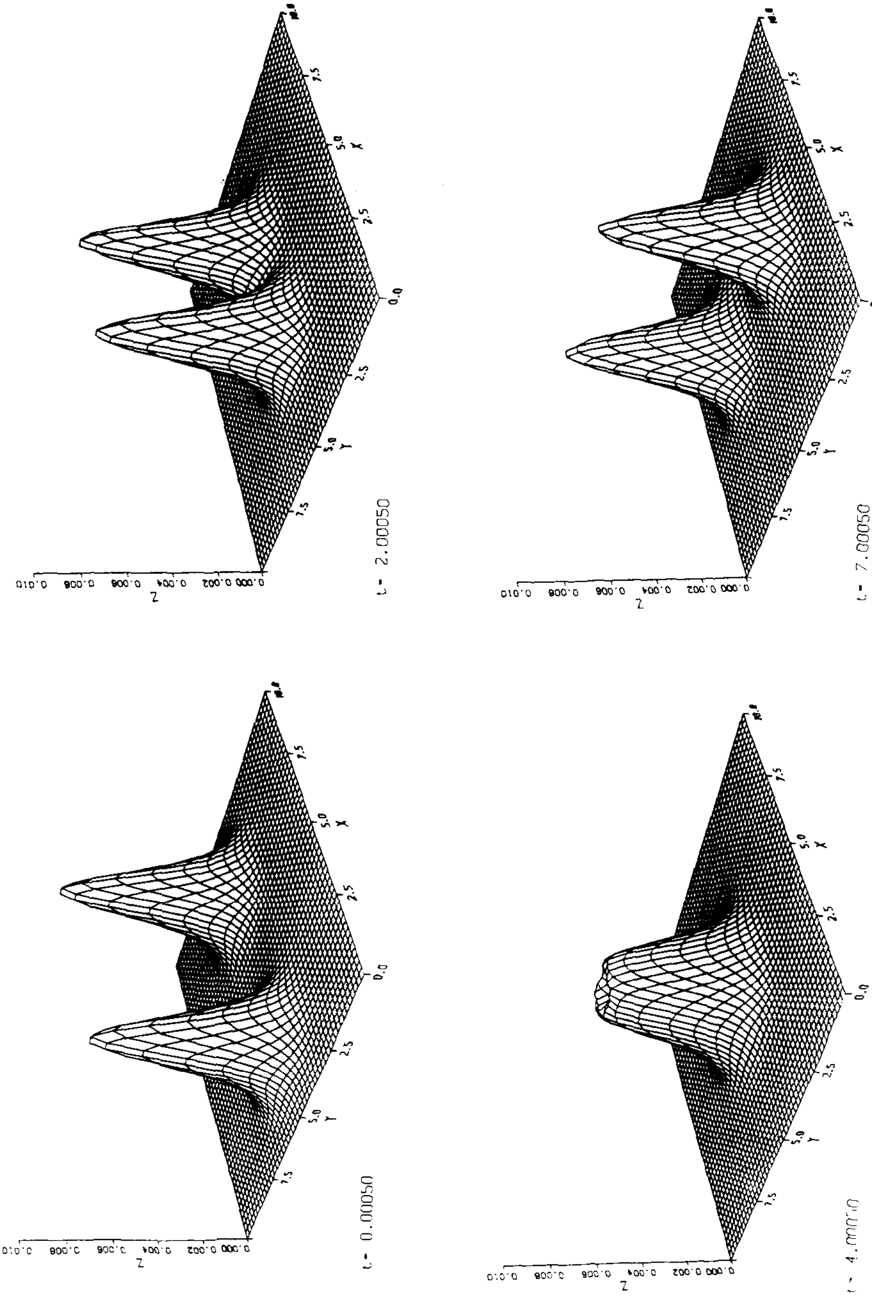

FIG. 1. Energy density of two critically coupled vortices as they collide and scatter at 90°.

the same as the two-dimensional problem, and the vortices will scatter at 90°. Then the only question is which halves of the strings go off with which vortices in this plane. It is easy to see that the energy of the system is less when two string ends on the same side of the vertical plane join together, which means that the strings intercommute.

We have also simulated the collision of a vortex with an anti-vortex, which represents two perfectly anti-parallel strings. As expected we find that the vortex and anti-vortex annihilate. This means that nearly anti-parallel strings will also intercommute. To see this imagine one of the strings in Fig. 2 turned around. When the strings collide the vortex/anti-vortex pair in the horizontal plane annihilate, so the strings break across this plane. The strings must therefore join across the vertical plane with the ends of the other string, and thus they intercommute.

It is not clear whether we can use these arguments for strings crossing at large angles, although we note that even when the strings cross perpendicularly they represent vortex/vortex scattering in the horizontal plane and vortex/anti-vortex scattering in the vertical plane, which should lead to intercommutation. To test this we have run a full three-dimensional simulation of two strings colliding perpendicular to each other. This is shown in Fig. 3, where the density of the distribution of points corresponds to the local energy density. It is easy to see that in the initial conditions the strings are isolated lines of high energy density. The strings

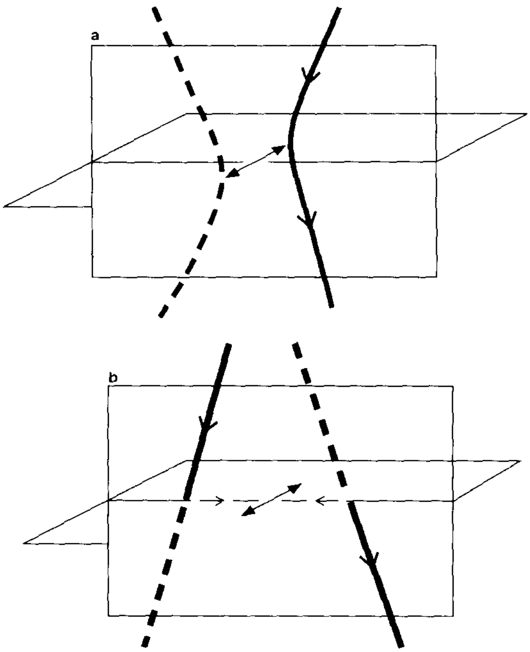

FIG. 2. Demonstration that 90° scattering of vortices leads to intercommutation of nearly parallel strings (see text): (a) before scatttering; (b) after scattering.

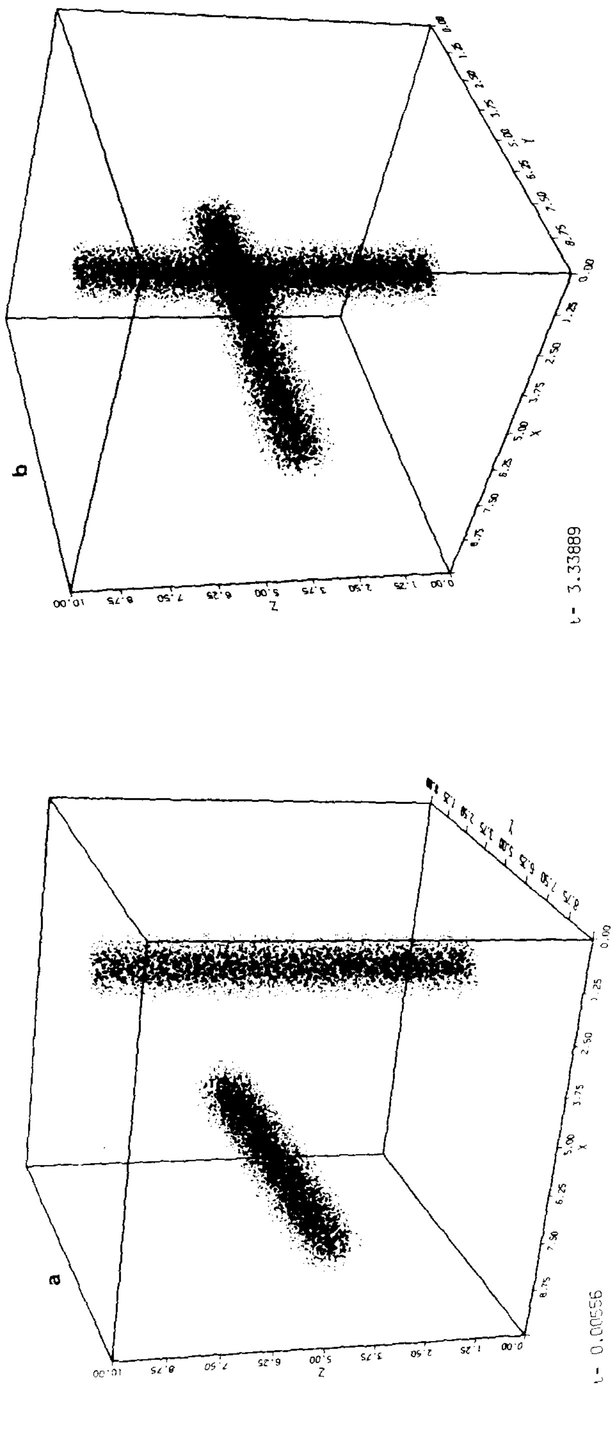

FIG. 3. *Intercommutation of two perpendicular strings.*

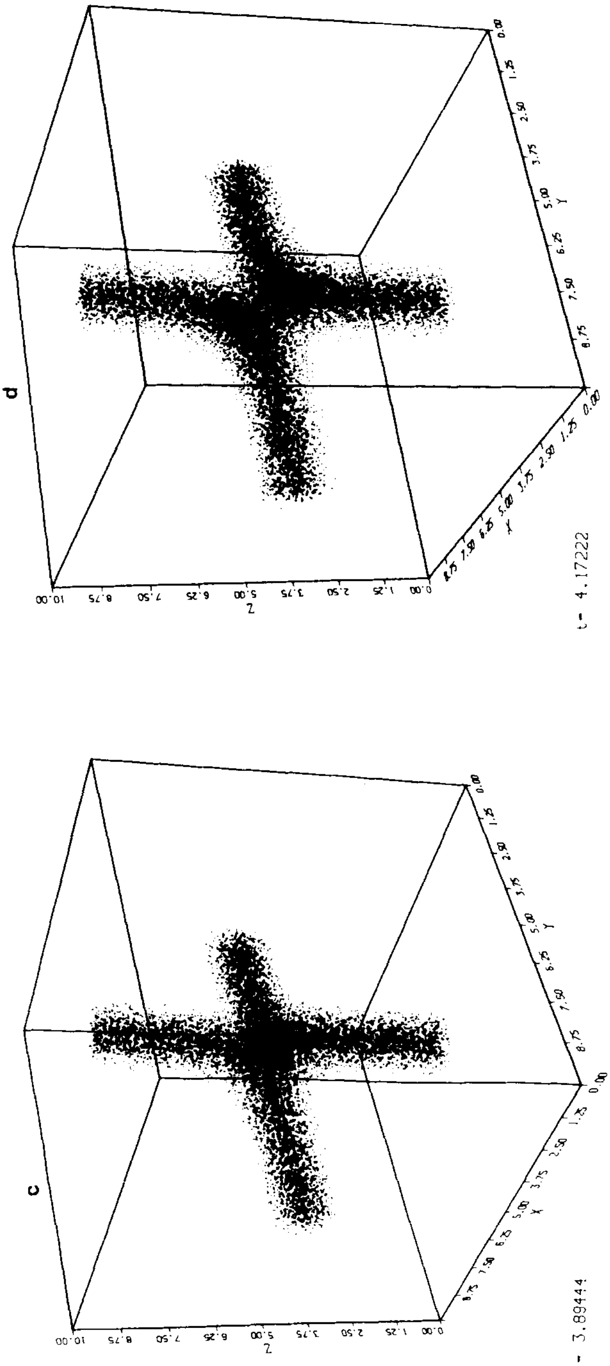

FIG. 3—*Continued*

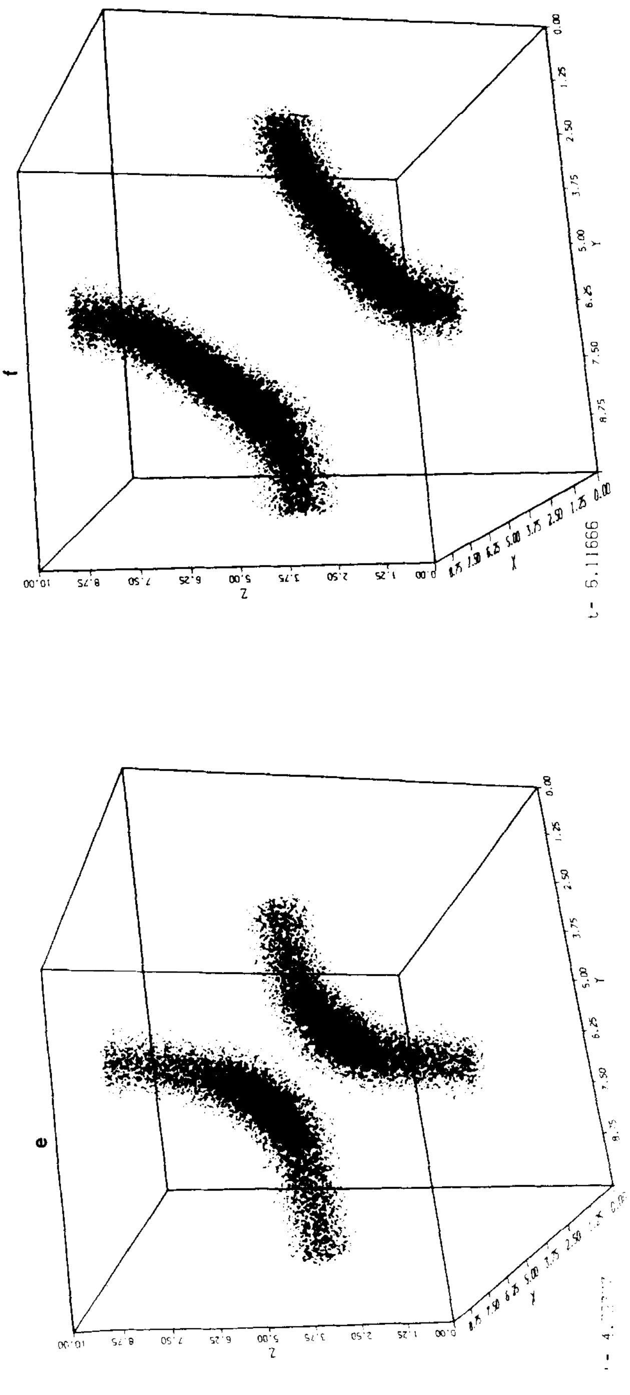

FIG. 3—*Continued*

move toward each other but do not interact until they are almost touching each other. When they are very close the sections of the strings nearest to each other can be seen to bulge out toward the other string, even though the coupling constant is at the critical value. Finally, it can be seen that the strings do indeed intercommute.

We have also observed that when a vortex and an anti-vortex collide with sufficient energy ($\beta > 0.9c$) they annihilate, but then a vortex/anti-vortex pair is re-created. What is even more interesting is that the vortex and anti-vortex go out in the direction in which the original vortex and anti-vortex came in—as if they have been scattered directly backwards (and not as if they had passed through each other). For nearly anti-parallel strings we still expect the strings will intercommute, because the original vortex/anti-vortex pair annihilate, but the subsequent production of another vortex/anti-vortex pair may be an indication of the formation of a small loop of string from the interaction. We are investigating this possibility further.

Acknowledgments

We would like to acknowledge useful conversations with Laurence Jacobs, So-Young Pi, Peter Ruback, Paul Shellard, and Terry Walker. We would like to thank Robert M. Price, Lawrence Perlman, and Gil Williams of Control Data Corporation and Lloyd M. Thorndyke and Carl S. Ledbetter of the ETA Systems, Inc. for their continued interest, support, and encouragement and access to the Scientific Information Services' CDC CYBER 205 at Kansas City, MO (Grant No. 13578), where some of these calculations were performed, the National Allocation Committee for the John von Neumann National Supercomputer Center for access to the two CDC CYBER 205s and the ETA-10 at JVNC (Grants 110128, 171812, 171813, 551701–551705), the Control Data Corporation PACER Fellowship grants (Grants 85PCR06, 86PCR01, and 88PCR01) for financial support, ETA Systems, Inc. for financial support (Grants 304658, 1312963, and 1333824), the Department of Energy (DOE Contract DE-AC02-86ER40284) for financial support, the Natural Sciences and Engineering Research Council of Canada (Grants NSERC A8420 and NSERC A9030] for financial support, Scotia High End Computing, Ltd. for financial support, Myrias Computer Corporation (Grant 028167) for financial support, Dalhousie University (Grant 19830682) for financial support, the Continuing Education Division, Technical University of Nova Scotia for financial support, Energy, Mines and Resources, Government of Canada (Grant 23313-7-0022/01SQ) for financial support, Atlantic Canada Opportunities Agency (AAP Project 2060-339, 905 and AAP Project 2060-343, 394) for financial support and the Canada/Nova Scotia Technology Transfer and Industrial Innovation Agreement (Grants 87TTII01 and 88TTII01) for further financial support. One of the authors (E. M.) would like to thank CDC/ETA Systems, Inc. for a grant which made his visit to Berkeley, California possible.

References

1. H. B. Nielsen and P. Olesen, *Nucl. Phys. B* **61**, 45 (1973).
2. A. Vilenkin, *Phys. Rep.* **121**, 203 (1985).
3. N. Turok, *Phys. Rev. Lett.* **55**, 1801 (1985).
4. T. Vachaspati and A. Vilenkin, *Phys. Rev. D* **30**, 2036 (1984).
5. K. J. M. Moriarty, E. Myers, and C. Rebbi, *Phys. Lett. B* **207**, 411 (1988); K. J. M. Moriarty, E. Myers, and C. Rebbi, "Interactions of Cosmic Strings," in *Proceedings of the Yale Workshop on Cosmic Strings*, edited by L. Krauss and F. Accetta (World Sci., Singapore, 1989), page 11.

6. K. J. M. MORIARTY, E. MYERS, AND C. REBBI, *Comput. Phys. Commun.* **54**, 273 (1989).
7. V. L. GINZBURG AND L. D. LANDAU, *Zh. Eksper. Teoret. Fiz.* **20**, 1064 (1950).
8. A. A. ABRIKOSOV, *Zh. Eksper. Teoret. Fiz.* **32**, 1442 (1957) (*Sov. Phys. JETP* **5**, 1174 (1957)); L. P. GORKOV, *Zh. Eksper. Teoret Fiz.* **34**, 734 (1958); **36**, 1918 (1959) (*Sov. Phys. JETP* **7**, 505 (1958); **9**, 1364 (1959)).
9. L. JACOBS AND C. REBBI, *Phys. Rev. B.* **19**, 4486 (1979).
10. For reviews see M. CREUTZ, L. JACOBS, AND C. REBBI, *Phys. Rep.* **95**, 201 (1983); J. KOGUT, *Rev. Mod. Phys.* **55**, 775 (1983).
11. P. RUBACK, *Nucl. Phys. B* **296**, 669 (1988).

RECEIVED: January 18, 1989; REVISED: April 25, 1989

K. J. M. MORIARTY

Department of Mathematics, Statistics and Computing Science
Dalhousie University, Halifax, Nova Scotia B3H 3J5 Canada;
Institute for Advanced Study
Princeton, New Jersey 08540;
John von Neumann National Supercomputer Center
665 College Road East
Princeton, New Jersey 08540

ERIC MYERS

Physics Department, Boston University
Boston, Massachusetts 02215;
Department of Mathematics, Statistics and Computing Science
Dalhousie University, Halifax, Nova Scotia B3H 3J5 Canada

CLAUDIO REBBI

Physics Department, Boston University
Boston, Massachusetts 02215